

**Upper critical field for underdoped high- $T_c$  superconductors: Pseudogap and stripe phase**

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We investigate the upper critical field in a stripe phase and in the presence of a phenomenological pseudogap. Our results indicate that the formation of stripes affects the Landau orbits and results in an enhancement of  $H_{c2}$ . On the other hand, a phenomenologically introduced pseudogap leads to a reduction of the upper critical field. This effect is of particular importance when the magnitude of the gap is of the order of the superconducting transition temperature. We have found that a suppression of the upper critical field takes place also for the gap that originates from the charge-density waves.

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**I. INTRODUCTION**

The high-temperature superconductors (HTSC) exhibit qualitative differences with respect to the classical superconducting systems. Normal-state properties of underdoped superconductors and the upward curvature of the upper critical field ( $H_{c2}$ ) belong to one of the most spectacular examples. The presence of a normal-state pseudogap has been confirmed with the help of different experimental techniques such as angle-resolved photoemission,<sup>1-3</sup> intrinsic tunneling spectroscopy,<sup>4,5</sup> NMR,<sup>6,7</sup> infrared,<sup>8</sup> and transport<sup>9</sup> measurements. Despite a wide spectrum of experimental data the underlying microscopic mechanism is far from being understood. A tempting hypothesis that the pseudogap is a precursor of the superconducting gap has not definitively been confirmed. In particular, the neutron-scattering experiments<sup>10</sup> reveal qualitative differences between the isotope effects observed for the superconductivity and the pseudogap. Moreover, results obtained with the help of intrinsic tunneling spectroscopy<sup>4,5</sup> speak against the superconducting origin of the pseudogap. The coexistence of superconductivity and the charge-density waves<sup>11-14</sup> can be considered as a possible scenario, that accounts for differences between superconductivity and the normal-state gap.

Inhomogeneous distribution of holes which enter the copper-oxygen planes in the doping process can give rise to the formation of stripes. This phase consists of antiferromagnetic domains which are separated by the hole-rich domain walls. Experimental evidence for the formation of stripes is material-dependent. Static stripes have been discovered in nickelates,  $\text{La}_2\text{NiO}_{4+\delta}$ ,<sup>15</sup>  $\text{La}_{2-x}\text{Sr}_x\text{NiO}_4$ ,<sup>16</sup> whose structure is fundamentally the same as that of La-based cuprates. Many experiments show the presence of similar, but dynamic, correlations in underdoped  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  (LSCO).<sup>17-21</sup> In LSCO doped with Nd or Eu there is a transition from low-temperature orthorhombic to the tetragonal phase. This involves distortions of  $\text{CuO}_2$  planes due to the tilt of  $\text{CuO}_6$  octahedra producing stripe pinning potential. It leads to static or quasistatic stripes in these compounds.<sup>22-24</sup> Some experimental data for other cuprates can also be viewed as the signature of dynamic stripes, e.g., angle-resolved photoemission measurements for  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  (BSCCO) Ref. 25 and inelastic neutron scattering for  $\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$  (YBCO).<sup>26</sup> The presence of in-

commensurate static charge ordering in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.35}$  has recently been reported in Ref. 27. Furthermore, tunneling experiments for BSCCO suggest that vortices can play the role of the stripe pinning centers, revealing the nature of the nearby ordered stripe phase.<sup>28</sup> There is also a convincing theoretical argumentation speaking in favor of the stripe phase<sup>29-32</sup> in strongly correlated systems.

Differences between the high-temperature superconductors and classical systems show also up in the magnetic properties. The high- $T_c$  compounds are characterized by large values of the upper critical field and its unusual temperature dependence. The resistivity measurements clearly indicate an upward curvature of the upper critical field with no evidence of saturation even at genuinely low temperatures.<sup>33,34</sup> These results remain in disagreement with the conventional, microscopic approach.<sup>35</sup> This discrepancy can be explained as a result of the Josephson tunneling between superconducting clusters<sup>36,37</sup> produced by a macroscopic phase separation.

Due to the complexity of the Gor'kov equations one usually assumes that the normal-state properties of the system under consideration can properly be described by three-<sup>38</sup> or two-dimensional<sup>39</sup> electron gas. Recently, we have proposed an approach that enables calculation of the upper critical field for a two-dimensional lattice gas.<sup>40,41</sup> This method allows one to derive  $H_{c2}$  in a similar way as one calculates the critical temperature in the standard BCS formalism. Therefore, any extension of the analysis of the upper critical field is rather straightforward. In the present paper we calculate the upper critical field in a system that exhibits some important properties of hole-doped cuprates: stripe phase and the presence of the pseudogap. In the latter case we discuss the coexistence of superconductivity and charge-density wave as well as a phenomenological pseudogap. We show that the anomalous properties of the high-temperature superconductors are reflected in the upper critical field.

**II.  $H_{c2}$  IN THE PRESENCE OF CHARGE-DENSITY WAVES**

We consider a two-dimensional square lattice immersed in a perpendicular uniform magnetic field of magnitude  $H_z$ . We assume the nearest neighbor pairing interaction  $H_V$  that is responsible for anisotropic superconductivity and the interaction term,  $H_{\text{CDW}}$ , which leads to the charge-density waves

(CDW). The relevant Hamiltonian reads<sup>40,41</sup>

$$H = H_0 + H_V + H_{\text{CDW}}, \quad (1)$$

where

$$H_0 = \sum_{\langle ij \rangle, \sigma} t_{ij}(\mathbf{A}) c_{i\sigma}^\dagger c_{j\sigma} + g \mu_B H_z \sum_i (c_{i\uparrow}^\dagger c_{i\uparrow} - c_{i\downarrow}^\dagger c_{i\downarrow}). \quad (2)$$

Here,  $c_{i\sigma}^\dagger$  annihilates (creates) an electron with spin  $\sigma$  at the lattice site  $i$ ,  $g$  is the gyromagnetic ratio and  $\mu_B$  is the Bohr magneton.  $t_{ij}(\mathbf{A})$  is the nearest-neighbor hopping integral that in the presence of the magnetic field acquires the Peierls phase factor<sup>42,40</sup>

$$t_{ij}(\mathbf{A}) = t \exp\left(\frac{ie}{\hbar c} \int_{\mathbf{R}_j}^{\mathbf{R}_i} \mathbf{A} \cdot d\mathbf{l}\right). \quad (3)$$

In the mean-field approximation the pairing interaction and the CDW coupling take on the form

$$H_V = -V \sum_{\langle ij \rangle} (c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger \Delta_{ij} + c_{i\downarrow} c_{j\uparrow} \Delta_{ij}^*), \quad (4)$$

$$H_{\text{CDW}} = -\delta_{\text{CDW}} \sum_{j\sigma} e^{i\mathbf{Q} \cdot \mathbf{R}_j} c_{j\sigma}^\dagger c_{j\sigma}, \quad (5)$$

where  $\Delta_{ij} = \langle c_{i\downarrow} c_{j\uparrow} - c_{i\uparrow} c_{j\downarrow} \rangle$  is the superconducting singlet order parameter and  $\delta_{\text{CDW}}$  represents the magnitude of the CDW gap. The complexity of calculations strongly depends on the CDW modulation vector  $\mathbf{Q}$ . For sake of simplicity, we consider a commensurate charge-density wave with  $\mathbf{Q} = (\pi, \pi)$ . This choice of the modulation vector results in the gap in the density of states that opens in the middle of the band (in our case at the Fermi level) independently on the magnitude of the external magnetic field. Since the pseudogap hardly depends on the magnitude of the magnetic field,<sup>5</sup>  $\delta_{\text{CDW}}$  will be taken as a model parameter.

In order to calculate the upper critical field we make use of the unitary transformation that diagonalizes the kinetic part of the Hamiltonian.<sup>40,41</sup> In the case of the Landau gauge,  $\mathbf{A} = H_z(0, x, 0)$ , this transformation is determined by a plane-wave function in  $y$  direction and an eigenfunction of the Harper equation,<sup>43</sup>

$$\begin{aligned} & g(\bar{k}, p, m+1) + 2 \cos(hm - pa) g(\bar{k}, p, m) + g(\bar{k}, p, m-1) \\ & = t^{-1} E(\bar{k}, p) g(\bar{k}, p, m). \end{aligned} \quad (6)$$

Here,  $m$  is an integer number that enumerates the lattice sites in  $x$  direction, whereas  $h$  is the reduced magnetic field,  $h = 2\pi\Phi/\Phi_0$ , that is expressed by the ratio of the flux  $\Phi$  through the lattice cell and the flux quantum  $\Phi_0$ .  $p$  is the wave vector in  $y$  direction and  $\bar{k}$  is an additional quantum number, that in the absence of the magnetic field is the wave vector in  $x$  direction. In the new basis the normal-state Hamiltonian takes on the form:

$$H_0 = \sum_{\bar{k}, p, \sigma} E_{\bar{k}p\sigma} a_{\bar{k}p\sigma}^\dagger a_{\bar{k}p\sigma}, \quad (7)$$

$$\begin{aligned} H_{\text{CDW}} &= -\delta_{\text{CDW}} \sum_{\bar{k}, \bar{l}, p, m, \sigma} g^*(\bar{k}, p + \pi, m) g(\bar{l}, p, m) \\ &\quad \times e^{i\pi m} a_{\bar{k}p+\pi, \sigma}^\dagger a_{\bar{l}p\sigma}, \end{aligned} \quad (8)$$

where

$$E_{\bar{k}p\sigma} = E(\bar{k}, p) + \sigma g \mu_B H_z. \quad (9)$$

One can prove that if  $E$  represents the eigenvalue of the Harper equation obtained for the wave-vector  $p$  then  $-E$  is one of the eigenvalues corresponding to  $p + \pi$ . It can also be shown that  $\tilde{g}(\bar{k}, p, m) = g(\bar{k}, p + \pi, m) \exp(i\pi m)$  represents an eigenfunction of the Harper equation calculated for momentum  $p$  with the eigenvalue  $-E(\bar{k}, p + \pi)$ . With the help of these relations one can obtain analytically the energy spectrum of the Hamiltonian in the normal state,  $H_0 + H_{\text{CDW}}$ , provided that the eigenvalues of the Harper equation are known. In particular, one can calculate the anomalous Green's functions which are related to the superconducting order parameter:

$$\begin{aligned} \langle\langle a_{\bar{l}p\uparrow} | a_{\bar{k}, -p\downarrow} \rangle\rangle &= -\sum_m [X_{\bar{l}k\bar{p}}^*(m) \Delta^x(m) \\ &\quad + Y_{\bar{l}k\bar{p}}^*(m) \Delta^y(m)] K_{\bar{l}k\bar{p}}(\omega), \end{aligned} \quad (10)$$

where

$$K_{\bar{l}k\bar{p}}(\omega) = \frac{(\omega + E_{\bar{l}p\uparrow})(\omega - E_{\bar{k}, -p\downarrow}) + \delta_{\text{CDW}}^2}{(\omega^2 - E_{\bar{l}p\uparrow}^2 - \delta_{\text{CDW}}^2)(\omega^2 - E_{\bar{k}, -p\downarrow}^2 - \delta_{\text{CDW}}^2)}. \quad (11)$$

$X$  and  $Y$  are determined by the solution of the Harper equation

$$\begin{aligned} X_{\bar{l}k\bar{p}}(m) &= g(\bar{l}, p, m) g(\bar{k}, -p, m+1) \\ &\quad + g(\bar{l}, p, m+1) g(\bar{k}, -p, m), \end{aligned} \quad (12)$$

$$Y_{\bar{l}k\bar{p}}(m) = 2 \cos(p) g(\bar{l}, p, m) g(\bar{k}, -p, m). \quad (13)$$

Due to the plane-wave behavior in  $y$  direction the superconducting order parameter depends explicitly only on the position in  $x$  direction (see Ref. 41 for the details)

$$\Delta^x(m) = \frac{V}{N} \sum_{\bar{l}k\bar{p}} X_{\bar{l}k\bar{p}}(m) \langle a_{\bar{k}, -p\downarrow} a_{\bar{l}p\uparrow} \rangle, \quad (14)$$

$$\Delta^y(m) = \frac{V}{N} \sum_{\bar{l}k\bar{p}} Y_{\bar{l}k\bar{p}}(m) \langle a_{\bar{k}, -p\downarrow} a_{\bar{l}p\uparrow} \rangle. \quad (15)$$

One can see that the impact of the charge-density waves on superconductivity is brought about only by the modification of  $K_{\bar{l}k\bar{p}}(\omega)$ . This quantity, when integrated over  $\omega$  with the Fermi function, gives the Cooper-pair susceptibility. Equations (10), (14), and (15) allow one to calculate the upper critical field. It is determined as the highest magnitude of the magnetic field for which there exists a nonzero solution for  $\Delta^x(m)$  and  $\Delta^y(m)$ .

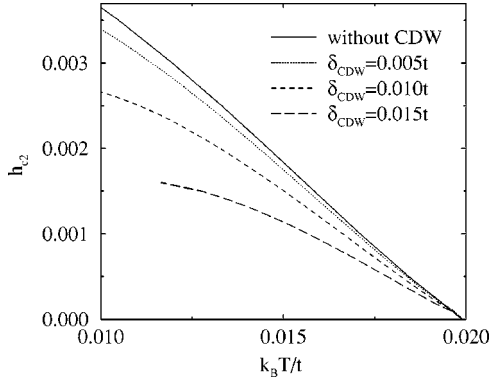


FIG. 1. Temperature dependence of the reduced upper critical field,  $h_{c2}$ , for different magnitudes of the CDW gap. We have used  $h_{c2} = (2\pi a^2/\Phi_0)H_{c2}$ , where  $a$  is the lattice constant.

Figure 1 shows the upper critical field, as a function of temperature calculated for different magnitudes of the CDW order parameter  $\delta_{\text{CDW}}$ . We have adjusted the magnitude of the pairing potential  $V$  that gives the same superconducting transition temperature  $kT_c = 0.02t$  for all values of  $\delta_{\text{CDW}}$ . These results have been obtained for  $120 \times 120$  cluster that at temperatures  $kT \sim 10^{-2}t$  gives convergent results (we refer to Ref. 41 for details of the cluster calculations). One can see that even for small magnitudes of the CDW order parameter the upper critical field is significantly reduced. However, qualitative temperature dependence of  $H_{c2}$  is not affected by the charge-density wave correlations. The reduction of the upper critical field due to the charge-density waves may be brought about by a direct coupling between CDW and superconducting order parameters as well as by the modification of the density of states. In order to distinguish these contributions we investigate the upper critical field in the presence of a phenomenological normal-state gap of arbitrary magnitude and depth. This problem will be discussed in the following section.

### III. $H_{c2}$ IN THE PRESENCE OF A PHENOMENOLOGICAL GAP

In this section we investigate the modification of the upper critical field that originates only from the normal-state gap in the density of states. In contradistinction to the analysis presented in the preceding section, the density of states may remain finite despite the presence of the gap. Here, the normal-state gap is characterized by the width  $2\delta$  and the relative depth  $(\rho_0 - \rho_{\text{PG}})/\rho_0$ , where  $\rho_{\text{PG}}$  and  $\rho_0$  denote the density of states in the presence and without the pseudogap, respectively. It is visualized in the inset of Fig. 2. In the absence of the CDW order the upper critical field is determined by Eq. (10) with

$$K_{\bar{k}p}(\omega) = \frac{1}{(\omega - E_{\bar{k}p\uparrow})(\omega + E_{\bar{k},-p\downarrow})}. \quad (16)$$

In order to account for the modification of the density of states we renormalize the normal-state propagators which give rise to the Cooper-pair susceptibility,

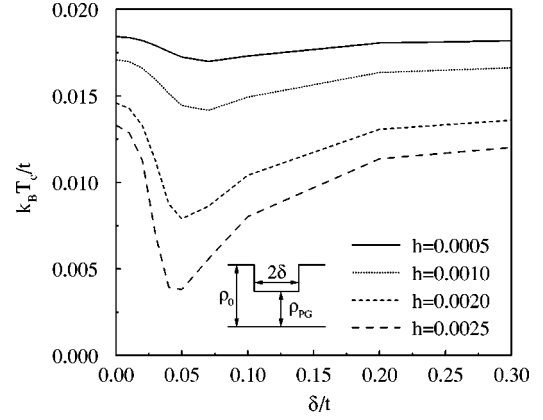


FIG. 2. Critical temperature as a function of the pseudogap width  $\delta$  for different values of the external magnetic field.  $\rho_{\text{PG}} = \frac{1}{2}\rho_0$  was used. The inset shows a schematic density of states in the vicinity of the Fermi level.

$$\begin{aligned} \frac{1}{\omega - E_{\bar{k}p\uparrow}} &\rightarrow \frac{\rho_{\text{PG}}}{\rho_0} \frac{1}{\omega - E_{\bar{k}p\uparrow}} + \frac{\rho_0 - \rho_{\text{PG}}}{2\rho_0} \\ &\times \left[ \left( 1 + \frac{E_{\bar{k}p\uparrow}}{\sqrt{E_{\bar{k}p\uparrow}^2 + \delta^2}} \right) \frac{1}{\omega - \sqrt{E_{\bar{k}p\uparrow}^2 + \delta^2}} \right. \\ &\left. + \left( 1 - \frac{E_{\bar{k}p\uparrow}}{\sqrt{E_{\bar{k}p\uparrow}^2 + \delta^2}} \right) \frac{1}{\omega + \sqrt{E_{\bar{k}p\uparrow}^2 + \delta^2}} \right]. \end{aligned} \quad (17)$$

In the limiting case  $\rho_{\text{PG}} = \rho_0$  one obtains the standard density of states as determined by the Hofstadter spectrum, whereas for  $\rho_{\text{PG}} = 0$  the density of states vanishes in the vicinity of the Fermi energy. Substituting the renormalized propagators into Eq. (16) one can calculate the upper critical field in the same way as described in the preceding section.

Figure 2 shows the superconducting transition temperature obtained for different values of the reduced magnetic field with  $\rho_{\text{PG}} = \frac{1}{2}\rho_0$ . As before, the intersite coupling  $V$  have been adjusted to obtain  $kT_c = 0.02t$  in the absence of magnetic field. One can see that the upper critical field is reduced due to the presence of the normal-state gap. The most significant lowering of  $H_{c2}$  takes place for finite values of the  $\delta$  which are comparable to the magnitude of the superconducting gap. This result originates from the fact that the Cooper-pair susceptibility is strongly peaked at the Fermi level with a characteristic energy scale that is determined by temperature. Therefore, for  $\delta \gg kT_c$  the pseudogap results in a global lowering of the density of states which can be compensated by an enhancement of the pairing potential. It means that assuming stronger pairing potential  $V$  we can reproduce  $H_{c2}(T)$  calculated in the absence of the gap.

We have found that the reduction of the upper critical field increases with the depth of the gap as depicted in Fig. 3. The inset of Fig. 3 shows a comparison of  $H_{c2}$  obtained for the phenomenological pseudogap with  $\rho_{\text{PG}} = 0$  and for the charge-density waves. A comparison of these results clearly

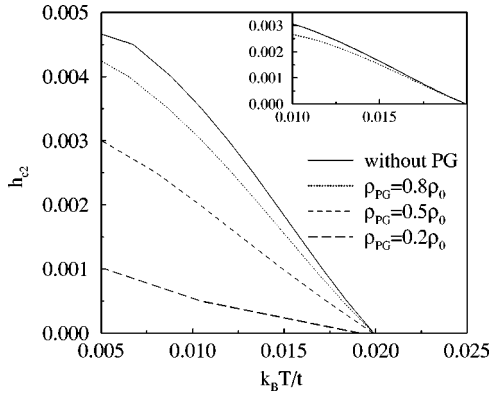


FIG. 3. Upper critical field as a function of temperature for different densities of states at the Fermi level. The half width of the pseudogap is  $\delta = 0.1t$ . The inset shows temperature dependence of upper critical field calculated for the phenomenological pseudogap with  $\rho_{PG} = 0$  (solid line) and for the CDW gap (dashed line). In both cases the half width is  $\delta = \delta_{CDW} = 0.01t$ .

indicate that the coupling between CDW and superconducting order parameters results in a small decrement of the upper critical field.

#### IV. $H_{c2}$ IN A STRIPE PHASE

Another unusual feature of HTSC, that we discuss in the present section, is related to inhomogeneous distribution of holes. It results in a stripe phase which consists of antiferromagnetic domains separated by hole-rich domain walls. We study how the upper critical field is affected by this specific distribution of carriers. In order to simulate the presence of a stripe phase we carry out the calculations for a long and narrow rectangular-shape clusters. We assume that the isolating, antiferromagnetic domains can be simulated by fixed boundary conditions in the direction perpendicular to the stripes (along the  $x$  axis). The spatial organization of the stripe structure has intensively been investigated on experimental<sup>22-24</sup> and theoretical grounds.<sup>29-32</sup> Experimental data for HTSC show that the width of stripes depends on the concentration of holes and is of the order of a few lattice constants. The neutron-scattering study of the stripe phase<sup>22</sup> suggests that the hole-rich domain walls are only single-cell wide. On the other hand, the numerical study of the two-dimensional  $t$ - $J$  model<sup>31</sup> shows that the domain walls may have a significant density of holes over three rows of sites. According to these results we consider  $150 \times n$  finite systems, where  $n = 2, 3$ , and  $7$ . Our simplified approach does not restore the actual structure of the stripe phase. In particular, for  $n = 1$  one obtains an unphysical, purely one-dimensional system, that hardly depends on the external magnetic field. Therefore, we investigate the rectangular-shape clusters with the width as a free parameter. Since we neglect the correlations between different stripes, the upper critical field is determined by Eqs. (10) and (16). We introduce stripes through the geometry of the system and specific boundary conditions. Therefore, our approach should be applicable to the cuprates with static or quasistatic (slowly fluctuating) stripes, e.g., for Nd- or Eu-doped LSCO. It could

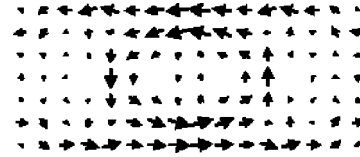


FIG. 4. Current distribution in a piece of stripe of width of seven sites, calculated for the reduced magnetic-field  $h = 0.1$ . The lengths of the arrows are proportional to the currents. Such a pattern is periodically repeated along the stripe. A slight deflection of currents at the edges of the figure originates from the superposition of the neighboring circulations.

also be appropriate for YBCO with static charge ordering.<sup>27</sup>

In the case of free-electron gas external magnetic field leads to the occurrence of rotationally invariant states corresponding to the Landau orbits. However, the geometry of the stripe phase may seriously affect the formation of the Landau orbits. This effect is of particular significance if the radii of the Landau orbits  $R_L$  exceed the width of the stripe  $an$  ( $a$  is the lattice constant). In order to visualize the impact of magnetic field on electrons in the stripe phase we have calculated the resulting current distribution. Within the framework of the linear-response theory the current operator is given by  $\hat{J}_l(x, y) = -\partial \hat{H} / \partial A_l(x, y)$ , where  $(x, y)$  denotes spatial coordinates and  $l$ 's are unit vectors in the lattice axes directions. Results obtained in the normal state ( $V = 0$ ) on a  $150 \times 7$  cluster with applied magnetic-field  $h = 0.1$  are presented in Fig. 4.

Modification of the Landau orbits affects the diamagnetic pair-breaking mechanism. Therefore, one may expect that superconductivity survives in the presence of much stronger magnetic fields than in the homogeneous phase. This observation is confirmed by the numerical calculations, as depicted in Fig. 5.

Here,  $150 \times 150$  cluster corresponds to an infinite system, whereas  $150 \times 7$ ,  $150 \times 3$ , and  $150 \times 2$  clusters correspond to stripes of the width of 6, 2, 1 lattice spacing, respectively. As we consider the anisotropic order parameter the  $150 \times 2$  clus-

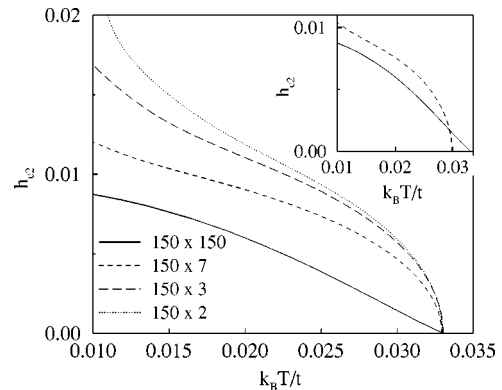


FIG. 5. Upper critical field as a function of temperature calculated for stripes of different width. We have chosen appropriate values of  $V$  which give the same transition temperature in the absence of magnetic field. The inset shows a comparison of upper critical field for  $150 \times 150$  and  $150 \times 7$  systems with the same value of the pairing strength  $V = 0.244t$ .

ter is already on the limit of applicability of our approach. The enhancement of  $H_{c2}$  is of particular importance for weak-magnetic fields, when  $R_L/na \rightarrow \infty$ . One can observe a dramatic change of the slope,  $dH_{c2}/dT$ , calculated at  $T = T_c$ . Here, the impact of the magnetic field on the superconducting transition temperature is much less than in the homogeneous two-dimensional case.

The pseudogap and stripes affect the superconducting properties of the system both in the presence and in the absence of the magnetic field. Modification of the density of states changes the effective coupling constant  $\lambda = \rho_{\text{FS}}V$  that enters the standard BCS gap equation. Therefore, we have directly compared the  $H_{c2}$  for systems, which in the absence of magnetic field are characterized by the same transition temperature (one can roughly say that  $\lambda = \text{const}$ ). In order to complete the discussion, we have also calculated the  $H_{c2}(T)$  for the case when the pairing potential does not depend on the pseudogap and the stripe structure ( $V = \text{const}$ ). Since, the opening of the pseudogap reduces  $T_c$  it results also in an additional decrement of the upper critical field, when compared to the results presented in Figs. 1–3. However, an enhancement of the  $H_{c2}$  in the stripe phase can take place despite the reduction of the superconducting transition temperature, as depicted in the inset of Fig. 5.

The coexistence of stripes and superconductivity has been established experimentally (see e.g., Ref. 24). However, up to our best knowledge, the upper critical field has not been measured in the stripe phase. According to our results such measurements should reveal a strong enhancement of the slope of  $H_{c2}(T)$  at  $T = T_c$ . When the temperature decreases this slope decreases, as well. This may effectively lead to a negative curvature of  $H_{c2}(T)$  close to  $T_c$ , despite the positive curvature in the homogeneous phase. The positive curvature may be restored at lower temperatures.

## V. CONCLUDING REMARKS

In order to clarify some physical aspects of our method one can compare it with approaches, which are commonly used to investigate  $H_{c2}$ . Previously, we have applied the same method to discuss the upper critical field for isotropic superconductivity.<sup>40,41</sup> Then, one ends up with the gap equation that can be written in the form

$$\Delta_i = \frac{V}{\beta} \sum_{j, \omega_n} \Delta_j G(i, j, \omega_n) G(i, j, -\omega_n). \quad (18)$$

Here,  $\Delta_i = \langle c_{i\downarrow} c_{i\uparrow} \rangle$  and  $G(i, j, \omega_n)$  is the one-electron Green's function in the presence of a uniform and static magnetic field. It is clear that the above equation is a lattice version of the linearized Gor'kov equations,<sup>35</sup> which determine the critical field at a second-order transition, where the

superconducting gap  $\Delta_i$  is vanishing.<sup>44</sup> However, our method does not allow to discuss the superconducting properties below the  $H_{c2}$  (e.g., the vortex state). In our approach the electron Green's functions have been calculated exactly, whereas in the standard case one makes use of the semiclassical approximation that neglects the Landau level quantization.

To conclude, we have investigated how the upper critical field is connected with different features of high-temperature superconductors. In particular, we have discussed  $H_{c2}$  in the presence of charge-density waves, phenomenological pseudogap and stripes. Our results suggest that a gap in the density of states reduces the upper critical field, independently on the underlying microscopic mechanism. For finite density of states at the Fermi level this reduction is mostly pronounced when the width of the gap is of the order of the superconducting transition temperature. In the phase with isotropic CDW gap the density of states at the Fermi level vanishes. Then, as one can expect, the upper critical field is strongly reduced even by a relatively small gap. Here, the coupling between the CDW and superconducting order parameters results in an additional reduction of  $H_{c2}$ . On the other hand, in the presence of stripes the upper critical field is enhanced, especially close to  $T_c$ . We attribute this effect to the reduction of the orbital pair-breaking mechanism since the radii of the Landau orbits are much larger than the width of the stripes.

The presented investigation of  $H_{c2}$  is restricted to the simplest case of the uniform magnetic field and neglects a possible disorder in the vortex system. It can originate from fluctuations close to the phase transition or inhomogeneous charge and spin distribution in the stripe phase. However, as we are concerned exclusively with the critical field at the second-order transition, these effects are of minor importance. We have also not discussed the reentrance of superconductivity in the strong-magnetic field. This effect has been investigated in the continuum model<sup>45,46</sup> as well as in the case of lattice gas,<sup>40,47</sup> when the structure of fractal energy spectrum is reflected in phase diagram. Theoretical argumentation that supports the reentrance of superconductivity remains valid also in the presence of pseudogap, at least on the simplest level that has been used in the present paper. However, in the genuinely strong-magnetic field the assumption of the field-independent gap is unphysical and microscopic investigation of this phenomenon is needed.

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